

Arithmetic Expression Evaluation Using Stacks and Queues

Suppose we only have 4 operators, +, -, ×, /, and parentheses, (and), in our arithmetic expressions. Their priorities in evaluating are given as follows:

$$\boxed{\mathbf{|}} \prec \boxed{(in,)} \prec \boxed{+, -} \prec \boxed{\times, /} \prec \boxed{(out)}$$

where **|** denotes the bottom of a stack (**vector**) or the end of a queue (**deque**), and “(in” denotes a “(” in the operator stack, and “(out” denotes a “(” outside the operator stack, i.e., a newly read “(” from the expression. We use one **deque** to maintain the token string (expression) and two **vectors** to store the intermediate values and operators. Only when the priority of the current operator from the token string is higher than the priority of the one on the top of the operator stack, we push back the current operator onto the stack. Otherwise, we evaluate the operation indicated by the top operator in the operator stack with appropriate arguments popped from the values stack, and push the result back to the value stack.

Consider the following scenario of evaluating the expression, $2/(3+2)+2\times 3$.

- | | | | | | | | | | | | | |
|---|-----|-----|-----|-------|---|---|---|-----|-----|------------|------------|--|
| <p>1. Exp: $2/(3+2)+2\times 3$
 Values: <table border="1" style="width: 100%;"><tr><td style="height: 20px;"> </td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="height: 20px;"> </td><td style="text-align: right;"> </td></tr></table></p> | | | | | <p>9. Exp: $+2\times 3$ match (and)
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">5</td><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">/</td><td style="text-align: right;"> </td></tr></table></p> | 5 | 2 | | / | | | |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| 5 | 2 | | | | | | | | | | | |
| / | | | | | | | | | | | | |
| <p>2. Exp: $/(3+2)+2\times 3$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="height: 20px;"> </td><td style="text-align: right;"> </td></tr></table></p> | 2 | | | | <p>10. Exp: $+2\times 3$ + \prec /, $2/5 \rightarrow 0.4$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="height: 20px;"> </td><td style="text-align: right;"> </td></tr></table></p> | 0.4 | | | | | | |
| 2 | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| 0.4 | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| <p>3. Exp: $(3+2)+2\times 3$ / \succ
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">/</td><td style="text-align: right;"> </td></tr></table></p> | 2 | | / | | <p>11. Exp: 2×3 + \succ
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">+</td><td style="text-align: right;"> </td></tr></table></p> | 0.4 | | + | | | | |
| 2 | | | | | | | | | | | | |
| / | | | | | | | | | | | | |
| 0.4 | | | | | | | | | | | | |
| + | | | | | | | | | | | | |
| <p>4. Exp: $3+2)+2\times 3$ (out \succ /
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">(/</td><td style="text-align: right;"> </td></tr></table></p> | 2 | | (/ | | <p>12. Exp: $\times 3$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">+</td><td style="text-align: right;"> </td></tr></table></p> | 2 | 0.4 | | + | | | |
| 2 | | | | | | | | | | | | |
| (/ | | | | | | | | | | | | |
| 2 | 0.4 | | | | | | | | | | | |
| + | | | | | | | | | | | | |
| <p>5. Exp: $+2)+2\times 3$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">3</td><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">(/</td><td style="text-align: right;"> </td></tr></table></p> | 3 | 2 | | (/ | | <p>13. Exp: 3 $\times \succ +$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">\times +</td><td style="text-align: right;"> </td></tr></table></p> | 2 | 0.4 | | \times + | | |
| 3 | 2 | | | | | | | | | | | |
| (/ | | | | | | | | | | | | |
| 2 | 0.4 | | | | | | | | | | | |
| \times + | | | | | | | | | | | | |
| <p>6. Exp: $2)+2\times 3$ + \succ (in
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">3</td><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">+ (/</td><td style="text-align: right;"> </td></tr></table></p> | 3 | 2 | | + (/ | | <p>14. Exp:
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">3</td><td style="text-align: right;">2</td><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">\times +</td><td style="text-align: right;"> </td></tr></table></p> | 3 | 2 | 0.4 | | \times + | |
| 3 | 2 | | | | | | | | | | | |
| + (/ | | | | | | | | | | | | |
| 3 | 2 | 0.4 | | | | | | | | | | |
| \times + | | | | | | | | | | | | |
| <p>7. Exp: $) + 2\times 3$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">2</td><td style="text-align: right;">3</td><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">+ (/</td><td style="text-align: right;"> </td></tr></table></p> | 2 | 3 | 2 | | + (/ | | <p>15. Exp: $\prec \times$, $2\times 3 \rightarrow 6$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">6</td><td style="text-align: right;">0.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">+</td><td style="text-align: right;"> </td></tr></table></p> | 6 | 0.4 | | + | |
| 2 | 3 | 2 | | | | | | | | | | |
| + (/ | | | | | | | | | | | | |
| 6 | 0.4 | | | | | | | | | | | |
| + | | | | | | | | | | | | |
| <p>8. Exp: $) + 2\times 3$) \prec +, $3+2 \rightarrow 5$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">5</td><td style="text-align: right;">2</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">(/</td><td style="text-align: right;"> </td></tr></table></p> | 5 | 2 | | (/ | | <p>16. Exp: \prec +, $0.4+6 \rightarrow 6.4$
 Values: <table border="1" style="width: 100%;"><tr><td style="text-align: right;">6.4</td><td style="text-align: right;"> </td></tr></table>
 Operators: <table border="1" style="width: 100%;"><tr><td style="height: 20px;"> </td><td style="text-align: right;"> </td></tr></table></p> | 6.4 | | | | | |
| 5 | 2 | | | | | | | | | | | |
| (/ | | | | | | | | | | | | |
| 6.4 | | | | | | | | | | | | |
| | | | | | | | | | | | | |